Delay-bounded Sink Mobility in Wireless Sensor Networks

Yu Gu*, Yusheng Ji*, Jie Li*, Biao Han*, and Baohua Zhao†‡
*Information Systems Architecture Science Research Division, National Institute of Informatics, Tokyo, Japan
†School of Computer Science, Univ. of Science and Technology of China, Hefei, Anhui, 230027, China
‡Department of Computer Science, University of Tsukuba, Hsukuba Science City, Ibaraki 305-8573, Japan

Abstract — This paper exploits sink mobility to prolong the network lifetime in wireless sensor networks (WSNs) where the information delay caused by moving the sink should be bounded. We build a unified framework for analyzing this joint sink mobility and routing problem. We offer a mathematical modeling that is general and captures diversified issues, e.g. sink mobility, routing, delay, etc. We discuss the induced subproblems and present efficient solutions for them. Then, we generalize these solutions and propose a polynomial-time optimal algorithm for the origin problem. In simulations, we show the benefits of involving a mobile sink. We also show that the impact of the delay bound on the network lifetime.

I. INTRODUCTION

Mobility management is one of the most important issues in wireless networks, and it has received extensive research efforts in different areas of wireless networks such as mobile ad hoc network (MANET), wireless mesh network (WMN), vehicular ad hoc network (VANET), etc.

Recently, there is a trend to investigate mobility as a means of relieving traffic burden and enhancing energy efficiency in wireless sensor network (WSN). We can classify sink mobility into two categories: random mobility and controlled mobility. Sinks in the first category move randomly within the network [1] [2]. Schemes based on random mobility are easy to implement, but they suffer from shortcomings like uncontrolled behaviors and poor performance. Recent research tends to use controlled mobility to improve the performance. The hardcore is how to jointly schedule different issues (e.g. sink mobility, data routing, information delay, etc.) to optimize the network lifetime.

For this paradigm, Gandham et al. first challenged this problem and proposed a heuristic algorithm [3]. Wang et al. relaxed the problem by doing the sink scheduling and data routing separately [4], and their proposed routing scheme can work only in a grid network topology. Guney et al. also studied the problem and proposed several greedy algorithms [5]. Recently, Shi et al. developed the first algorithm with performance guarantee [6]. Their works are quite enlightening. In our recent research [7], we proposed a column generation based algorithm that provides near-optimal performance.

In above proposals, they assume that sinks are high-speed so that information delay caused by moving the sink can be ignored. However, on the other hand, mobile sinks in physical worlds usually have limited speed. On the other hand, underlay applications like the real-time surveillance demand a delay upper bound. Therefore, it is natural to take delay issue into consideration.

Basagni et al. jointly considered the sink mobility and delay issue in [8]. But they assumed that the routes are predetermined. Wang et al. used multiple controllable sinks to travel among event locations to efficiently gather data. They considered issues like the mobile distance of a sink and time delay [9]. However, only one-hop routing has been used. Recently, Yun et al. [10] jointly considered the multi-hop routing, sink mobility and delay bound to improve the energy efficacy. However, the network model is totally different with ours. They still used the fast mobility assumption [13], so the delay is caused by nodes holding their transmissions until the location of the sink is most favorable for energy saving, not by the movement of the sink.

In this paper, we study delay-bounded sink mobility problem (DeSM) in WSNs. We assume that WSNs are deployed to monitor the surrounding environment and the data generation rate of sensors can be estimated accurately. We constrain the mobile sink to a set of sink sites. First, we propose a unified framework that covers most of the joint sink mobility, data routing and delay issue strategies. Based on this framework, we develop a mathematical formulation that is general and captures different issues. However, this formulation is a mixed integer nonlinear programming (MINLP) problem and is time-consuming to solve directly [11]. Therefore, instead of tackling the MINLP directly, we first discuss several induced subproblems, e.g. subproblems with zero/infinite delay bound or connected sink sites (sink sites are connected if for any two sites there exists a path that connects them and each edge of that path meets the delay constraint). We show that these subproblems are tractable and present optimal algorithms for them. Then, we generalize these solutions and propose a polynomial-time optimal approach for the origin DeSM problem. We study the benefits of using a mobile sink in simulations. We also show that the impact of the delay bound on the network lifetime.

Our main contributions are the following:
1) We provide a unified formulation of DeSM, which is general and practical.
2) We discuss subproblems of DeSM and offer efficient algorithms for them to guide the design of our algorithm.
TABLE I
THE POWER CONSUMPTION OF TWO RADIO MODULES IN WSNs [15]

<table>
<thead>
<tr>
<th>Chips</th>
<th>Transmission (mW)</th>
<th>Receive (mW)</th>
<th>Sleep (μW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC1000</td>
<td>19.8-80.1</td>
<td>22.2</td>
<td>0.6</td>
</tr>
<tr>
<td>CC2420</td>
<td>25.5-92.2</td>
<td>59.1</td>
<td>60</td>
</tr>
</tbody>
</table>

for the origin DeSM.
3) We generalize algorithms for subproblems and present a polynomial-time optimal algorithm for the DeSM.

The rest of the paper is organized as follows. We describe and formulate the DeSM problem in Section II. In Section III, we present induced subproblems. In Section IV, we propose the optimal algorithm for DeSM. Simulations are reported in Section V. Finally, Section VI concludes the paper.

II. DELAY-BOUNDED SINK MOBILITY PROBLEM

A. Network Model

We model a WSN as a graph $G = (V \cup V_0, L \cup L_0)$, where $V$ and $V_0$ is the set of sensors and sink sites. $L = \{V \times V\}$ is the set of wireless links between sensors. $l_{ij}$ is 1, if sensor $j$ is within the communication range ($r_i$) of sensor $i$, otherwise $l_{ij} = 0$. Similarly, $L_0 = \{V \times V_0\}$ is the set of potential links between sensors and sink sites. The sink has a maximum speed $V_{max}$. We assume that while the sink is moving, sensors will buffer their newly generated data, as in [10]. This could potentially cause a high delay for data packets. To bound such delay, a delay bound (i.e. $\tau$) should be set according to the underlay applications.

As in most previous proposals [3] [4] [12] [13], we do not explicitly consider radio interferences, i.e., we assume the data generation rates of sensors can be properly scaled so that underlay MACs like TDMA [14] can eliminate the interference among communications.

B. Energy Model

For a node $i$, we assume its data generation rate $\lambda_i$ can be estimated accurately. Its initial energy is $E_i$. The total energy consumption of $i$ cannot exceed $E_i$. Typically, the radio module is the most energy-consuming part, and thus its energy consumption consists of three parts: transmission, receive and sleep. Table I shows the power characteristics of two representative radio modules that have been widely used in wireless sensor platforms. Since usually the power assumption in sleep state is several orders of magnitude lower than in other states, it has nonsignificant impact on the network lifetime and thus can be ignored.

The energy cost for transmitting one unit data from node $i$ to $j$ (or to $s_0$ at site $k$) can be determined as follows [10],

$$e_{ij(k)} = \alpha + \beta \cdot d(i, j(k))^{\theta}$$ (1)

where $\alpha$ and $\beta$ are constant coefficients, $d(i, j(k))$ is the distance between node $i$ and node $j$ (or sink site $k$), and $\theta$ is the path loss index, which is typically in the range of 2 to 6 depending on the environment [16].

We denote energy cost for receiving one unit data as $e^R$, which is constant. Hence, energy consumption at node $i$ is

$$e_i = e_{ij}^T \left( \sum_{l_{ij} \in L} f_{ij} \right) + e_{ik}^T \left( \sum_{l_{ik} \in L_0} f_{ik} \right) + e^R \left( \sum_{l_{ji} \in L} f_{ji} \right)$$ (2)

where $f_{ij(k)}$ is the amount of data from node $i$ to node $j$ ($s_0$ in site $k$).
For the sink $s_0$, we assume it has limitless energy compared to sensor nodes.

The network lifetime is defined as:

Definition 1: Network Lifetime ($T$). The network lifetime ($T$) is defined as the elapsed time since the launch of this network till the instant that the first node dies.

Thus, based on the network and energy models, the problem of our concern can be stated as:

Problem Statement: Delay-bounded Sink Mobility Problem (DeSM). Given network topology $G$, maximize network lifetime by jointly sink scheduling and data routing subject to energy and delay constraints.

C. Problem Formulation

The network lifetime $T$ can be divided into epoches with different lengths: A new epoch begins when $s_0$ changes its location. $t_p$ is defined as the time span for that epoch. To connect $s_0$ with sink sites in the $p$th epoch, we define $x^p_k$ as the indicator:

$$x^p_k = \begin{cases} 1, & s_0 \text{ stays at site } k \text{ in } p\text{th epoch} \\ 0, & \text{otherwise} \end{cases}$$ (3)

For $s_0$, it keeps static during any epoch, therefore we have,

$$\sum_{k \in V_0} x^p_k = 1, \forall p$$ (4)

For the $p$th epoch, node $i$ should fulfill the flow constraint, i.e. the outgoing flow equals the incoming flow plus data it generated during the sink movement between $(p-1)$th and $p$th epoch and in the $p$th epoch:

$$\sum_{l_{ji} \in L} f^p_{ji} + (\Delta t_p + t_p) \cdot \lambda_i = \sum_{l_{ij} \in L} f^p_{ij} + \sum_{k \in V_0} x^p_k \cdot f^p_{ik}$$ (5)

where $f^p_{ij(k)}$ represents the amount of data from node $i$ to $j$ (site $k$) in the $p$th epoch, and $\Delta t_p$ is the travel time of $s_0$ during $(p-1)$th and $p$th epoch. It can be calculated as follows,

$$\Delta t_p = \begin{cases} 0, & \sum_{k_1, k_2 \in V_0} d(k_1, k_2) x^p_{k_1} x^p_{k_2}^{p-1} \leq V_{max}, \forall p = 0 \\ \sum_{k_1, k_2 \in V_0} d(k_1, k_2) x^p_{k_1} x^p_{k_2}^{p-1}, & \text{otherwise} \end{cases}$$ (6)

According to the delay constraint, $\Delta t_p$ should be less than or equal to $\tau$, which is far less than $t_p$ naturally.

$$\Delta t_p \leq \tau \ll t_p$$ (7)

Given the flow over node $i$ in $p$th epoch, its energy consumption is:
sink problem can be modeled as follows, the WSN. We name this subproblem as the Z-DeSM problem.

A. Zero Delay

scheduling data routing when

from its neighbors plus data generated by itself.

subject to

The above linear programming (LP) is referred to as ZERO hereinafter.

Theorem 2 guarantees that INFI is the optimal solution for the I-DeSM problem.

The above linear programming (LP) is referred to as ZERO hereinafter.

Eq. (16) ensures that for each sensor, the total energy consumption would not exceed its initial energy.

For the complexity, we have the following conclusion,

Theorem 1. For the Z-DeSM, it can be solved optimally in $O(mn^6)$ time.

Proof. For the Z-DeSM, we can apply ZERO to all sink sites and pick up the one with the longest lifetime as the site for $s_0$. Considering ZERO is a LP problem and can be solved optimally in $O(n^6)$ time [17], the overall computational complexity is $O(m \cdot n^6) = O(mn^6)$. $\diamond$End$\diamond$

B. Infinite Speed

In this part, we study the subproblem of DeSM where the sink can move at a high speed (approximately infinite). This assumption has been widely used in previous proposals [18] [13] [7]. In this case, $s_0$ can move freely among sink sites during the operation of the WSN. We name this subproblem as the I-DeSM problem.

For I-DeSM, as proved in [6] and our previous paper [7] (from different perspectives), we have the following conclusion,

Lemma 1. Given a solution for I-DeSM, as long as the sojourn time assigned for the sink site remains the same, the network lifetime remains the same regardless of the ordering and frequency of sink sites’ presence.

Basically, Lemma 1 states that, since $s_0$ can move at a very high speed, the travel route doesn’t impact the optimal network lifetime as long as the sojourn time $s_0$ stays at some sink site remains the same. Therefore, define $t_k$ as the sojourn time $s_0$ stays in site $k$, $f_{ij}^k$ as the sojourn flow from node $i$ to $j$ when $s_0$ stays at site $k$ and $f_{ik}$ as the sojourn flow from node $i$ to $s_0$ which stays at site $k$. The I-DeSM problem can also be solved optimally in polynomial-time by the following LP formulation,

(INFI) Max$(T = \sum_k t_k)$ subject to

$$\sum_{l_i \in E} f_{ij}^k + t_k \cdot \lambda_i = \sum_{l_i \in E} f_{ij} + f_{ik}, \forall i, k \tag{15}$$

$$\sum_{k \in V}(e_{ij}^T(l_i \in E) + e_{ik}^T(l_i \in E) + e_R(l_i \in E) f_{ij}) \leq E_i \tag{16}$$

$$t_k > 0, f_{ij}^k > 0, \forall i \in V, k \in V_0, \forall p \tag{17}$$

Theorem 2. For the I-DeSM, it can be solved optimally in $O(m^3n^6)$ time by the INFI optimization.

Proof. Lemma 1 guarantees that INFI is the optimal solution for the I-DeSM. For INFI, it is a LP problem with $(m+mn^2)$ variables and can be solved optimally in $O(m^3n^6)$ time [17], the overall computational complexity is $O(m^3n^6)$. $\diamond$End$\diamond$
Algorithm 1: The Sink-schedule-flow-rebuild algorithm

Input: $\omega = \{T = \sum_k t_k, f_{ij}, f_{ik}\}$; 
$\overline{P} = \{\pi(1) \to \cdots \pi(p) \to \pi(|P|)\}$ 

Output: $\omega' = \{T' = \sum_p t_p, f_{ij}', f_{ik}'\}$ 

begin 
  for $k = 1; k \leq m; k + +$ do 
    for $\pi(p) = k$ do 
      /*pause time assignment for pth epoch (a.k.a.*/ 
      on $\pi(p)$ on $\overline{P}$ */ 
      $t_p = \sum_{\pi(p') = k} \Delta t_p' - \Delta t_p$ 
    for $i \in V$ do 
      /*flow rebuild for i in the pth epoch */ 
      $f_{ij}' = f_{ij}' + \Delta t_p + t_p$, $\forall j$ 
      /*flow rebuild for k in the pth epoch */ 
      $f_{ik}' = f_{ik}' + \Delta t_p + t_p$, $\forall k$ 
  end 
end

C. Connected Sink Sites

In this part, we study the subproblem of DeSM where the sink sites are connected. In this case, sink sites form a graph,

Definition 2: Sink Site Graph. A sink site graph is an undirected graph $G' = \{V_0; L' = V_0 \times V_0\}$ with a link $l_{k_1,k_2}$ ($l_{k_1,k_2} \in L'$) indicating that the distance between them is no longer than $V_{max} \cdot \tau$.

We name the induced subproblem where the sink site graph is connected as C-DeSM. For C-DeSM, we develop an optimal Sink-Scheduling and Data-Routing approach (SSDR), which runs in polynomial-time. The SSDR approach can be summarized as following three steps,

Step1. Run the INFI optimization to get the optimal solution $\omega = \{T = \sum_k t_k, f_{ij}, f_{ik}\}$:

1) $T_k$: sojourn time $s_0$ stays at site $k$; $T = \sum_k T_k$;
2) $f_{ij}'$: flow from node $i$ to $j$ when $s_0$ stays at site $k$;
3) $f_{ik}'$: flow from node $i$ to $s_0$ staying at site $k$.

Step2. Run the depth-first search (DFS) to find a path on the sink site graph $G'$ that includes all the sink sites (may visit some site multiple times).

4) $\overline{P} = \{\pi(1) \to \cdots \pi(p) \to \pi(|P|)\}$, where $\pi(p) \in V_0$

Step3. Run the The Sink-schedule-flow-rebuild algorithm (as shown in Alg.1) to assign the visit time of every point on the path found in Step2 and corresponding routes based on the data obtained in Step1.

In the above approach, we divide C-DeSM into two parts and conquer them one-by-one. In Step1, we ignore the path $s_0$ would travel and use INFI to get the sojourn time $s_0$ stays at site $k$ and corresponding routes. Then, we randomly select a path which contains all the sink sites, which is easy since $G'$ is connected. In the final step, which is the key step, we assign the visit time of every point on that path and corresponding flows. For the approach, we can prove the following theorem,

Theorem 3. The SSDR approach is an optimal algorithm for C-DeSM and can be solved in $O(m^3n^6)$ time.

Proof. Omitted due to page limit.

From Step2, we know that,

Corollary 1. For a given instance of C-DeSM, $\tau$ doesn’t impact on the network lifetime as long as $G'$ is connected.

In the next section, we will extend the SSDR to solve the origin DeSM problem.

IV. Extended SSDR Algorithm for DeSM

To solve the origin DeSM problem, we prove the following conclusion,

Lemma 2. For an instance of DeSM, if its sink site graph $G'$ is not connected, we can divide $G'$ into connected subgraphs, each of which can be solved optimally by the SSDR. The overall optimal solution for this instance is the same solution of the subgraph with the longest network lifetime.

Proof. Due to page limit, we only give a sketch here.

The proof is based on contradiction. Assume that for an instance of DeSM, we have an optimal solution which involves two sites from two different subgraphs. This means that we find a sink path including these two sites that meets the delay constraint. Thus these two sites are connected and should be in the same subgraph. This is the contradiction and finishes the proof. End

Based on Lemma 2, we propose an extended SSDR approach (E-SSDR) to solve the origin DeSM optimally:

Step1. Divide $G'$ into connected subgraphs.

Step2. Apply the SSDR approach to each subgraphs and obtains the optimal sink path as well as corresponding routes.

Step3. Choose the solution of the subgraph with the longest network lifetime as the output.

For the E-SSDR approach, we have the following conclusion.

Theorem 4. For the DeSM, it can be solved optimally in $O(m^3n^6)$ time by the E-SSDR approach.

Proof. It is straightforward that E-SSDR is optimal for the DeSM. The complexity of E-SSDR is decided by Step2, which is $O(m^3n^6)$. End

V. A Case Study

As shown in Fig.1(a), 10 sensors and 3 sink sites are randomly distributed in a square area ($100 \times 100$). Network parameters are shown in Tab.II.

First, we set $\tau = 0$ and it becomes one instance of the Z-DeSM problem. The optimal solution for Z-DeSM is shown in Fig.1(b), where $s_0$ stays at sink site o3 for 58.52 unit time (e.g. minutes or hours). The sensor that dies first is $s_0$, because it has to relay data for $s_1 - s_5$.
Then, we set $V_{max} = \infty$ and it becomes one instance of the I-DeSM problem. The optimal solution found by ZERO is shown in Fig.1(c), where the optimal solution contains two epochs, each is activating for 30.3 unit time. In the first epoch, the sink resides in $o_3$ and the green lines show the data flows. Since $s_6$ relays data for $s_1$ to $s_5$, after the first epoch, its remaining energy is the smallest (454.6). Then, in the second epoch, the sink resides in $o_1$ and the red lines are data flows. In this epoch, $s_1$ relays data for all other 9 sensors and $s_6$ relays for $s_7$ to $s_{10}$. After $p_2$, the remaining energy for all 10 sensors is $< 0.181.9, 263.8, 0, 0.91, 818.2, 636.4, 818.2, 636.4 >$.

Then, we fix $V_{max}$ to 100 and change $\tau$. The result is shown in Fig.1(d). Since $d(o_1, o_2) \approx 109$, $d(o_1, o_3) \approx 93$, $d(o_2, o_3) \approx 26$, when $\tau \cdot V_{max}$ varies from 0 to 25 all three sites are disconnected with each other, therefore the optimal solution is 58.52. When $\tau \cdot V_{max}$ varies from 26 to 92, $o_2$ and $o_3$ are connected as a subgraph while $o_1$ is the other subgraph. But, as we analyzed, $s_6$ is the bottleneck sensor when $s_0$ resides in $o_2$ or $o_3$. Therefore, though the subgraph constituted by $o_2$ and $o_3$ beats the other subgraph, the network lifetime remains the same, i.e. 58.52. When $\tau \cdot V_{max}$ increases from 93, all three sites are connected, and the network lifetime increases to 60.6. Therefore, we conclude that $\tau$ does impact on the network lifetime, but not in a continuous way.

Further simulations confirmed the observations of both case studies. We omit details due to page limit.

VI. CONCLUSION

In this paper, we proposed a unified framework to analyze the sink mobility problem in WSNs with delay constraint. We presented a mathematical formulation that jointly considers different issues such as sink scheduling, data routing, bounded delay, etc. The formulation is general and can be extended. However, this formulation is a mixed integer nonlinear programming and is time-consuming to solve directly. Therefore we discussed several induced subproblems and developed corresponding optimal algorithms. Then, we generalized these solutions and proposed a polynomial-time optimal approach for the origin problem. We studied the benefits of using a mobile sink in simulations. We also showed that the impact of the delay bound on the network lifetime.

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